

Application of mathematics in Economics: A study with different perspectives

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Abstract

Every academic subjects have its own standards by which it judges the merits of what researchers claim to be true. In the study of Physical Sciences it typically requires experimental verification. Whereas in the study of History it requires links to the original sources related to the past. In Economics there are two purposes to apply mathematics, one is the mathematical tools needed to make and understand economic arguments, the second one is, to make you comfortable talking about economics using the shorthand of mathematics. The major development of the second quarter of 20th century in the field of economics was the mathematization of economics. The application of mathematical techniques to the analysis of economic problems is a methodological possibility. This technique often called as Mathematical economics. This Mathematical economics is the application of mathematical methods to represent theories and analyze problems in economics.

During the last six decades, this methodological option has been linked to history of a significant part of economic analysis. Mathematics allows economists to form meaningful, testable propositions about wide-ranging and complex subjects which could less easily be expressed informally.

The purpose of this course is to provide a comprehensive exposition of basic mathematical instruments that are commonly used in all fields in economics - microeconomics, macroeconomics, econometrics, international trade and finance, public finance, money and banking. The methodology of the study is based on secondary data. The uses of Differential calculus, Maxima and Minima etc., in economics are attempted to study in this article. The major finding of the study is mathematics played a pivotal role in economic theory and research.

Keywords: mathematical, wide-ranging

Introduction

Economics is a social science. It does not just describe what goes on in the economy. It attempts to explain how the economy operates and to make predictions about what may happen to specified economic variables if certain changes take place, e.g. what effect a crop failure will have on crop prices, what effect a given increase in sales tax will have on the price of finished goods, what will happen to unemployment if government expenditure is increased. It also suggests some guidelines that firms, governments or other economic agents might follow if they wished to allocate resources efficiently. Mathematics is fundamental to any serious application of economics to these areas. Applications of Mathematics in Economics presents an overview of the (qualitative and graphical) methods and perspectives of economists. It provides concrete form to economic laws and relationships and made more practical. Use of Mathematics helps in systematic understanding of the relationship and in derivation of certain results which would either be impossible through verbal argument, or would involve complex, tedious and difficult processes. Mathematics now a day a very important tool used in economic analysis. Use of mathematics gives a better understanding about different economic conditions. Out of vast applications of mathematics in economics, in this study we will try to find out the different possibilities of applications. We will check the relation between both subjects. Though they are completely different subjects but after this study we will able to understand how they are inter related with each other.

Objectives of the Study

1. To study the application of Mathematics in Economic theory concepts.
2. To observe the different use of Mathematical tools in the research of economics.

Methodology

The study is based on secondary data. The author refers the research reports, articles, Books, Journals and Websites. The use of Mathematical techniques and tools in economic theoretical concepts are explained through illustrations

Some Application of Mathematics in Economics:

1. Functions

A mathematical function describes the relation between two or more than two variables. That is, a function expresses dependence of one variable on one or more other variables. Thus, if the value of a variable y depends on another variable x , mathematically we may write:

$$y=f(x)$$

The above expression implies that every value of the variable y is determined by a unique value of the variable x . In the function (1) y is known as the dependent variable and x is the independent variable.

In economics Demand is a function of price and production is a function of factors of production. In usual language we say that demand (D) depends on the price, in mathematical terms

we would say that demand is function of price while in symbolic notations we would write

$$D=f(P)$$

This functional relationship is a mathematical concept. Similarly, supply function of a commodity X is expressed as:

$$S = f(P)$$

Using the graph we can show the demand and supply curve as follows.

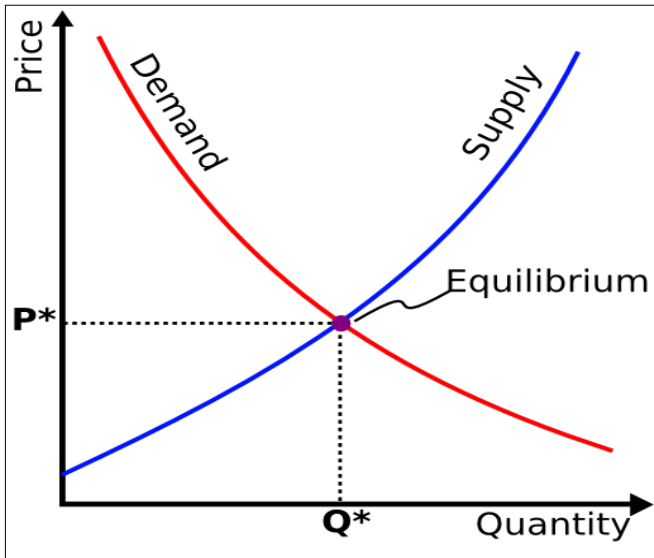


Fig 1

Likewise, utility, cost, Revenue, profit, supply, savings etc., are the functions of some related variables. If the utility of a commodity (U) depends on the quantity of the commodity consumed or used (q), we write,

$$U=f(Q)$$

When the value of the variable Y depends on more than two variables X_1, X_2, \dots, X_n this function is written in general form as:

$$Y = f(X_1, X_2, X_3, X_4, \dots, X_n)$$

Similarly in the case of production function, one variable is determined by a group of variable means one dependent variable is depends on group of independent variables.

$$Q_x=f(P_x, P_y, P_z, \dots)$$

2. Straight Line

Linear function is a important mathematical concept. The linear function($ax + b$ where a, b in set of real numbers) is usually represented in a graph as a straight line. This function is also used in economic analysis, especially in demand and supply analysis. For Example: the demand curve under perfect competition is a straight line, which can be expressed as ‘Linear Equation’ The demand can also write as

$$D=f(P) \text{ \& } D=7-P.$$

Here ‘P’ is the independent variable and ‘D’ is dependent variable, and with a unit fall in price, demand rises by a unit. Straight line depreciation method charges cost evenly throughout the useful life of a fixed asset. This depreciation method is appropriate where economic benefits from an asset are expected to be realized evenly over its useful life. Straight line method is also convenient to use where no reliable estimate can be made regarding the pattern of economic benefits expected to be derived over an asset's useful life. Straight line description can be calculated using of the following formulas

Table 1

1) Depreciation per annum	=	$\frac{(\text{Cost} - \text{Residual Value})}{\text{Useful Life}}$
2) Depreciation per annum	=	$(\text{Cost} - \text{Residual Value}) \times \text{Rate of depreciation}$

3. Parabola

Quadratic Function or second Degree function is yet another mathematical concept. We can define a quadratic function as a Quadratic function is a function in which the highest power of x is 2. There may also be a term in x and a constant, but no other terms. The graph of this function is a “parabola” i.e U shaped.

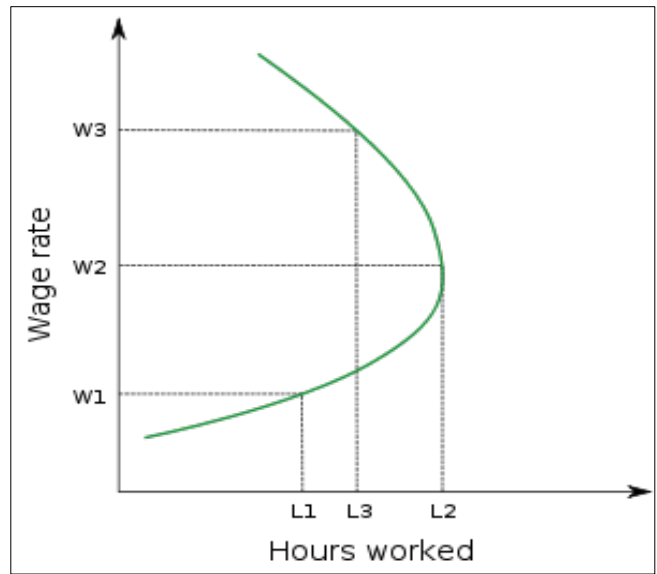


Fig 2

This technique is applied in Economics in cost “functions” since, cost curves in economics are U shaped. Quadratic functions are often used in economics to represent both the production cost function and the revenue function. Suppose that the cost C, in rupees of producing x mobile phones is given by

$$C(x) = 400 + 8x + 0.1x^2$$

4. Differentiation

Most of the economic decisions are based on mathematical concepts “Derivatives” this process is called “marginal analysis”. In mathematics derivative means rate of change of certain object with respect to another object. The concept of “margin “is a basic concept in economics. You are always differentiating to find ‘marginals’. The concept of ‘marginals’

(marginal revenue, marginal product, marginal cost) etc is about the most important concept in microeconomics, because all decisions are taken 'at the margin'. If the production is more your marginal revenue(MR) will fall and marginal cos(MC)t will rise so the profit can be maximized by producing where MR = MC. Graphically MC can be represented as follows

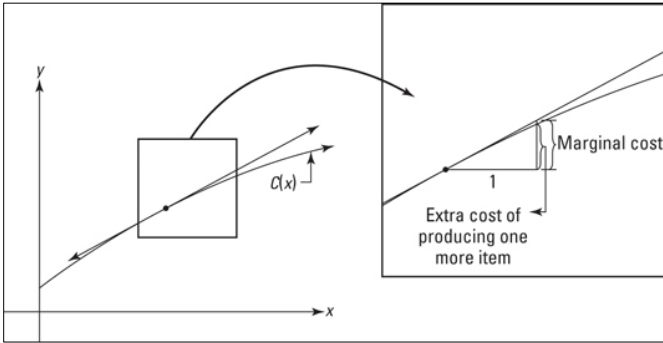


Fig 3

Similarly if the total utility function $U = f(Q)$ Then the marginal utility is the first older derivative of the total utility function. i. e du/dq . Hence all marginal concepts such as marginal productivity, marginal revenue, marginal cost, marginal rate of substitution (MRS), marginal propensity to consume (MPC), marginal propensity to save (MPS) are the first older derivatives of the relevant functions. in short, Differentiation is helpful to derive the marginal functions from the total functions.

5. Slope

Graphically the value of dy/dx is the slope or gradient of a curve. The concept of slope is important in economics because it is used to measure the rate at which changes are taking place. Slope means that a unit change in x , the independent variable will result in a change in y by the amount of b . slope = change in y /change in x = rise/run. Slope shows both steepness and direction. This technique is used in economics, to find the 'slope' of the curves like demand curves, revenue curves, cost curves, indifference curves and isoquants. When the slope is negative, then the curve will be a falling curve otherwise if the slope is positive, then the curve will be a rising one. Following figure gives a graphical example of slop used in economic concepts.

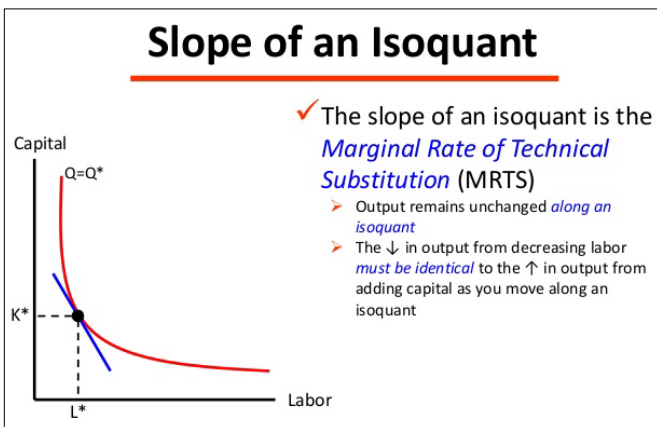


Fig 4

6. Euler's theorem or partial differentiation

If $Z = f(x,y)$ is a homogeneous function of degree 'x' then, $x.dz/dx+y.dz/dy=xZ$

Example if $U = x^3 + y^3 + z^3 - 3xyz$ prove that $x.du/dx+y.du/dy+z.du/dz=3u$.

This is known as "Euler's Theorem on homogeneous functions. This Euler's theorem can be applied to Marginal productivity theory of Distribution in Economics.

If $z=f(L, K)$ is linearly homogeneous function, $z=L.dz/dl+ k.dz/dk$

Hence Partial derivatives play a prominent role in economics, in which most functions describing economic behavior posit that the behavior depends on more than one variable.

Another example is a societal consumption function may describe the amount spent on consumer goods as depending on both income and wealth; the marginal propensity to consume is then the partial derivative of the consumption function with respect to income.

7. Maxima & Minima

Let $f(x)$ be a real valued function defined on an interval I . Then, $f(x)$ is said to have the maximum value in the interval I , if there exists a point a in I such that $f(x) \leq f(a)$ for all $x \in I$. The number $f(a)$ is called the maxima or the maximum value of $f(x)$ in the interval I and the point a is called a point of maxima of f in the interval I .

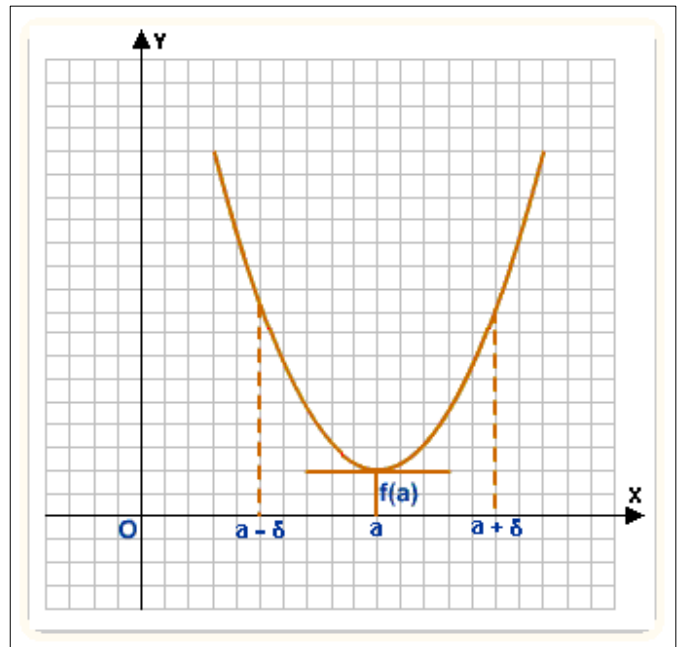


Fig 5

Let $f(x)$ be a real valued function defined on an interval I . Then, $f(x)$ is said to have the maximum value in the interval I , if there exists a point a in I such that $f(x) \leq f(a)$ for all $x \in I$. The number $f(a)$ is called the maxima or the maximum value of $f(x)$ in the interval I and the point a is called a point of maxima of f in the interval I .

Let $f(x)$ be a real valued function defined on an interval I . Then $f(x)$ is said to have the minimum value in interval I , if there exists a point $a \in I$ such that $f(x) \geq f(a)$ for all $x \in I$. The number $f(a)$ is called the minima or minimum value of $f(x)$ in

the interval I and the point a is called a point of minima of f in the interval I.

In economics, we are interested in analyzing the consumer equilibrium and Firm's equilibrium. The optimization in Calculus is helpful in the study of Consumer's equilibrium and Equilibrium of the Firm. Consumer is in equilibrium only when his utility is maximum. Hence, with the help of optimization technique, we can calculate the maximum utility of the consumer and in turn the consumer's equilibrium. Likewise, firm is in equilibrium only when its profit is maximum. Through differentiation, we can determine the maximum profit of the firm, and in turn, Equilibrium of the Firm.

Symbolically, we Can represent this as $\pi = R - C$. Where, π = profit, R=Revenue, C=Cost. For maximum profit $\partial\pi/\partial q = 0$ and $\partial^2 \pi / \partial q^2 < 0$. Where q= the level of output.

Sometimes, the firm's objective may be to maximize the cost for a given level of output. Minimization in Mathematics is useful to calculate the minimum average cost and minimum marginal cost a given level of output.

For a maximum, First derivative=0, second derivative 0

8. Difference and Differential Equations

The theory of differential equations has become an essential tool of economic analysis particularly since computer has become commonly available. It would be difficult to comprehend the contemporary literature of economics if one does not understand basic concepts (such as bifurcations and

chaos) and results of modern theory of differential equations. Difference and Differential equations are very helpful to study the "Macro Economic Theories" and the "Theories of Economic Growth." Application of "Difference Equations" to economic theories are abundant. A few of them are Multiplier and Accelerator Interaction and Cob-web Model. Likewise, the application of differential equations to economic analysis is also much. For instance, A differential equation expresses the rate of change of the current state as a function of the current state. A simple illustration of this type of dependence is changes of the Gross Domestic Product (GDP) over time. Consider state x of the GDP of the economy. The rate of change of the GDP is proportional to the current GDP $x'(t) = gx(t)$, where t stands for time and x'(t) the derivative of the function x with respect to t. The growth rate of the GDP is $x'(t)/x$. If the growth rate g is given at any time t, the GDP at t is given by solving the differential equation. The solution is $x(t) = x(0)e^{gt}$. The solution tells that the GDP decays (increases) exponentially in time when g is negative (positive)

9. Elasticity

The theory of elasticity refers to the responsiveness of supply and demand to changes in price. The product means that any change in price can result in changes in supply or demand. The inelastic product means that changes in price do not affect to a noticeable degree, the supply or demand.

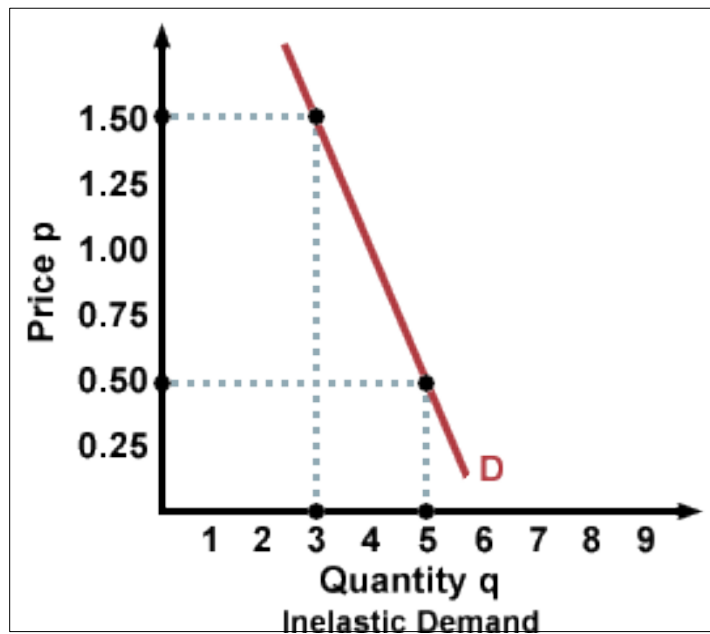


Fig 6

Demand is a decreasing function of price and the "Elasticity of demand" is defined as the ratio of proportionate change in demand to proportionate change in price. In fact, elasticity is a mathematical concept. symbolically we can represent the elasticity of demand as ϵ or $\epsilon_d = dq/dp \cdot P/Q$ Where P price and Q is quantity demanded of a commodity.

10. Simultaneous Equations

The equilibrium price is that price at which the quantity demanded equals the quantity supplied. This analysis is carried

out through the solution of a system of two Simultaneous Equations with two unknowns: namely price and quantity.

Example: Demand function = $9P + 20$

Supply function = $11P + 14$

$$9P - 11p = 14 - 20$$

$$-2P = -6 \quad P = 6/2 = 3.$$

Following figure shows the graphical representation of perfect demand and supply function in the form of a system of simultaneous equation

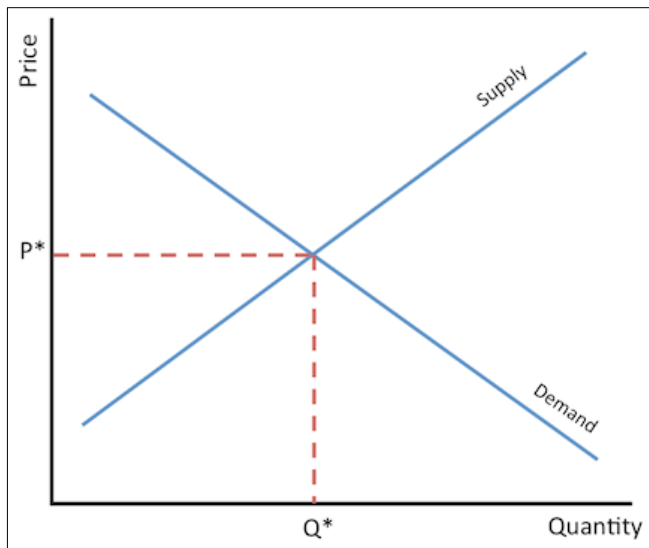


Fig 7

11. Economic Models

prof. P.C Mahalanob is, famous mathematician and economist has given a good model to economy in the second five-year plan. In planning models, sectoral targets are fixed only with the help of mathematical models like input –output model and linear programming. Determinants and matrix Algebra of Mathematics are of immense use in such techniques. Mathematics is indispensable to calculate “capital formation” and interest rates. thus, in all most all fields of economics, mathematics is useful.

12. Input-Output Analysis

Input-output analysis ("I-O") is a form of economic analysis based on the interdependencies between economic sectors. This method is most commonly used for estimating the impacts of positive or negative economic shocks and analyzing the ripple effects throughout an economy.

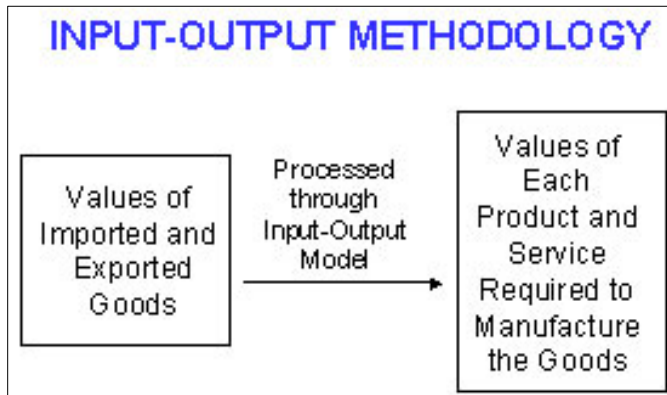


Fig 8

Input-output analysis is a mathematical technique for studying the production structure of an economy on the assumption of mutual interdependence of the various sectors of the economy. The primary purpose of the input-output analysis is to calculate the out-put levels in various industries that would be required by particular levels of demand for final goods.

Conclusion

Hence from the above study we can conclude that economical concepts are incomplete without the use of mathematics. To understand economics properly we need to use mathematics in every point. With the use of mathematical techniques, now a days economical concepts are understood in such a way that one can grow their interest in the subject. Hence both the subjects are very much inter related with each other and use of mathematics cannot be denied in economic analysis.

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