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Public debt and economic growth in Nigeria: A dynamic threshold regression analysis

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Abstract
This study uses annual data from 1981 to 2019 to evaluate the ideal public debt threshold that would be consistent with Nigeria's long-term growth goal. It showed overall support for an inverted U-Shaped relationship between the three categories of public debt—domestic, external, and total—and economic growth. For total public debt as a percentage of GDP, the result indicates a threshold level of 41 % (as the optimal debt benchmark). The implication of this finding is that debt accumulation in excess of the estimated threshold levels could hurt economic growth. Against the general notion, the study found no support for external debt accumulation opportunities. The policy import of our result is, therefore, the need for government to exercise caution in further debt accumulation, the absence of such a caution in debt accumulation could result in debt levels that are inconsistent with the country's growth objective.
JEL Classification: F34, E62, H62, H63.

Keywords: Economic growth, dynamic threshold regression, public debt, Nigeria

Introduction

Debt can function as a two-aged sword (Hu, 2019; Chechetti *et al*., 2011) [8]. When debt is low, increase its increase could encourage economic growth; however, when debt reaches some threshold value, further increase could lower GDP growth. Debt is known to help bridge the government-financing gap. Yet, economists – as noted - worry when debt level becomes quite large relative to GDP because this would imply that the higher tax rates required to meet the debt interest burden - or debt service cost - can produce some disincentive effect and lead to debt overhang. This has led a country such as the United Kingdom to enact legislations - e. g., the UK code for fiscal stability enacted in 1998 and more recently the 2002 stability and growth pact (SGP) - that constrains the government from borrowing outside sustainable (debt) benchmark or limits. In WAMZ, there are also benchmark on debt in line with the Macroeconomic convergence criteria (Onye and Umoh, 2021; Tarawalie, Sissoho, Conte and Ahotor, 2013; 2014) [12, 20, 21]

Indeed, the justification for public borrowing derives from the neoclassical growth model which prescribes that capital scarce countries have to borrow to increase their capital accumulation and steady-state level of output per capita. Yet the global economic and financial crisis has provided further rationalization - particularly for the developing countries - to borrow as they are often confronted with the need to increase expenditure levels and raise capital inflows (Omotosho, Bawa, and Doguwa 2016; Greenidge *et al*, 2012) [7, 11]. This is evident in countries such as Portugal, Italy, Cyprus, Greece, Belgium, and the US which recorded a debt/GDP ratio of over 100 percent in 2015. In Japan, the debt/GDP ratio was as high as 250 percent during the same period, 2015 (IMF, 2016) and this has continued to generate curiosity among researchers and policymakers regarding the level of debt accumulation that is optimal for economic growth. By 2020 Japan has emerged among the top-three most indebted country in the world and yet the Asian country remains economically stable and creditworthy. This is because Japan's national assets are clearly visible to the world economies and therefore Japan's public debt is asset-based and asset-backed. For Nigeria, however, notwithstanding the country's apparent comparative low debt profile, i.e., in relation to the GDP and in comparison with sister countries, there is serious concern regarding the country's debt sustainability and fiscal consolidation effort (see Panel 2, Fig 1).

As noted, the accumulation of debt beyond certain limits could offset the positive impacts of public borrowing, as typified by the Euro-crisis. Yet, the debate on the growth implications of debt accumulation is an ongoing one, and results from empirical works are still inconclusive. While a strand of the literature postulates that high levels of debt are associated with a large negative effect on growth, other strands of literature predict a positive effect of high debt on growth. For Nigeria, while the study by Ezeabasili *et al* (2011) and Boboye and Ojo (2012) [1] showed that external debt confer negative effects on economic growth, Sulaiman and Azeez (2012) found empirical support for a positive relationship. In their seminal work in this regard, Reinhart and Rogoff (2010) [16] document evidence of a significant threshold effect in the relationship between debt and economic prosperity.

Nonetheless, there are dissenting views regarding the debt threshold hypothesis. Those who hold opposing views argue that there is no debt threshold above which debt could necessarily constrain output growth. These scholars, instead, raise endogeneity concerns and argued that weak growth is the cause of high levels of debt. The proponents of this view further argue that - priority should be on increasing growth rather than reducing debt - and that foreign borrowing has a net positive effect on output and income as long as the net inflow of borrowed funds exceeds interest payments, and the marginal productivity of investment is greater than the rate of interest on the debt (Green and Kahn, 1990). In other words, foreign borrowing is at optimal up to the point where the marginal productivity of capital or investment equals the rate of interest on external debt. In Nigeria - however as

with most other countries with mounting debts and debt service costs - the persistence of rising debt service (on public) debt calls to question the empirical merit of this hypothesis.

Notwithstanding the two opposing viewpoints, economists and policymakers seem to have reached a consensus that excessive debt can cause negative growth effects and macroeconomic distortions through debt overhang and crowding-out effects. This is despite the two opposing views regarding the impact of debt accumulation on economic growth. Therefore, the critical task is that of determining the threshold (inflection) point in the debt-growth relationship.

The objective of this study is to investigate the threshold point, if any, beyond which public debt could begin to hamper Nigeria's economic growth prospects. The findings are expected to guide policy in the design of an optimal public debt framework that is conducive for Nigeria's economic growth objectives. To the best of our knowledge, scanty studies had focused on the determination of the threshold point beyond which the incurrence of additional debt hampers economic growth in Nigeria, especially following the rebasing of the country's GDP from 1990 base year to 2010 and the debt cancellation in 2005.

The remainder of the paper is organized as follows. Section 2 present the stylized characterization of Nigeria's debt profile while section 3 reviews relevant literature. The result of the empirical isometric is presented and discussed in section 4 while section 5 concludes the paper with some policy implications.

1. Stylized Characterization of Nigeria's Public Debt and Debt Sustainability Profile

1.1. Nigeria's Public Debt Profile

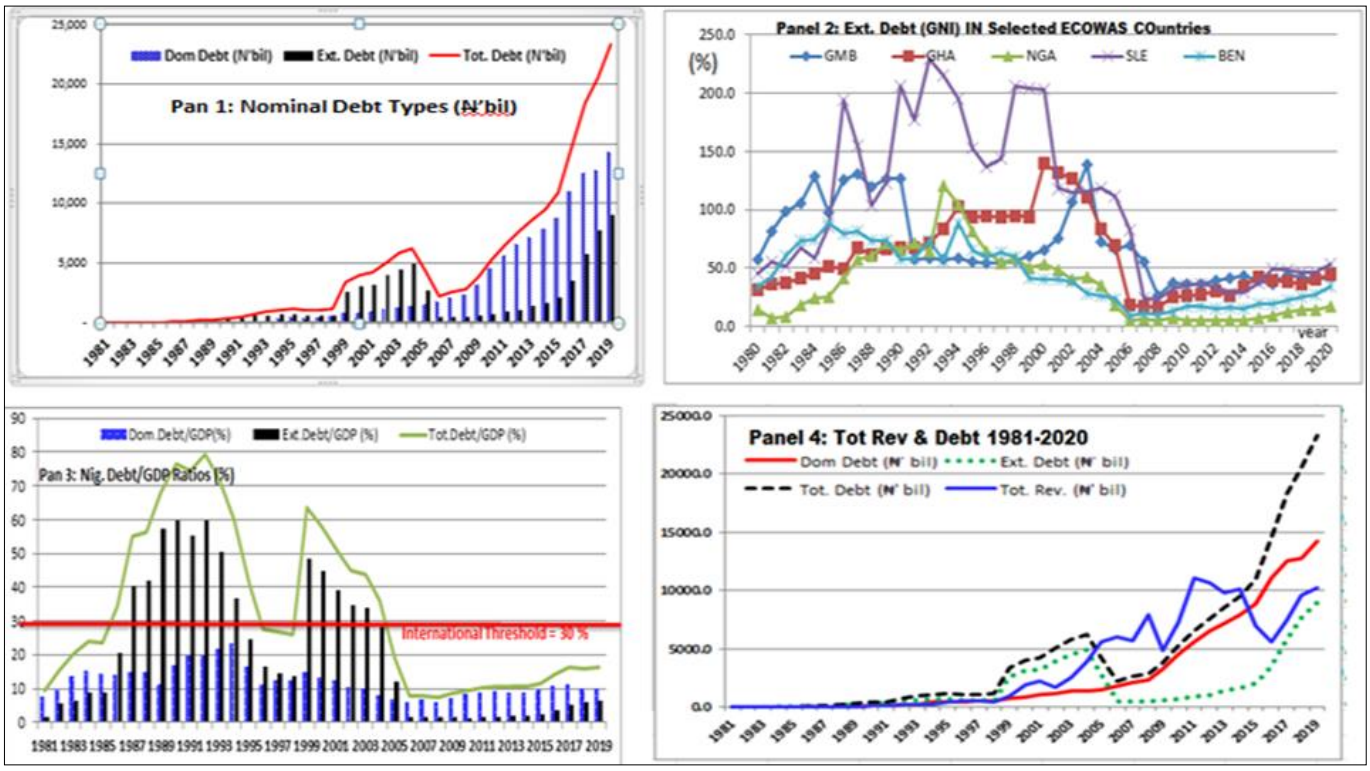
As with other developing countries, Nigeria has relied on both external and domestic borrowing to support the financing of its developmental goals over the years. Nigeria's external debt, for instance, was N 4.9 trillion in 2004 from N 3.9 trillion in 2002 before the debt forgiveness in 2005. Nigeria secured external debt relief from the Paris Club in 2005, leading to the cancellation of about 60 per cent of the US$30.85 billion being owed by the country. The debt relief negotiation was largely motivated by the need to free up resources for investment and faster economic growth in the country. In the aftermath of the 2005 debt forgiveness, Nigeria's external debt plummeted as low as 0.5 trillion in 2006 and 0.43 trillion in 2007. By 2013, Nigeria's external debt has risen to 1.6 trillion. More recently, total external debt has more than doubled its pre-2005 value with foreign dent hitting N 3.5 trillion in 2016, N 5.8 trillion in 2017, N 7.8 trillion in 2018, and N 9.0 trillion.

As with the nascent rising debt profile, Nigeria's (external) debt service rose persistently in the aftermath of the debt relief obtained in 2005. Following the 2005 debt forgiveness, Nigeria's debt service cost (on external debt) declined from $US8.8 billion in 2005 to $US 1.0 billion in 2007. However, by 2014, debt service cost has rather reversed as it rose to $US4.5 billion. Modest decline was recorded in 2015 and 2016 with the country posting debt service costs of $US 1.6 billion and $US 2.5 billion respectively. More recently, Nigeria's debt service has rather rose to a whopping $US 3.5 billion in 2017, $US 5.4 billion in 2018, and $US 5.1 billion in 2019 (see Table 1).

Table 1: Debt and Debt Sustainability Macroeconomic Variables (2002-2019)

Year	Tot. Dom Debt (N'bil)	Tot. Ext Debt (N'bil)	Tot Debt (N'bil)	Nom GDP (N'bil)	Rev (Tot Fed.Col Rev (N 'bil)	Debt Serv. (N'bil)	Gen Govt. Exp (N'bil)	Gen Govt Rev. (N'bil)	Fisc. Def. (N'bil)	Dom. Debt/GDP (%)	Debt Serv ($US 'bil)	Ext. Debt/ GDP (%)	Tot Debt/ GDP (%)	Tot Debt/ Rev (%)
2002	1166.0	3932.9	5098.9	11332.3	1731.8	178.7	2196.3	2348.5	-152.1	10.289	1.5	34.7052	44.9944	294.42
2003	1329.7	4478.3	5808.0	13301.6	2575.1	211.1	3086.8	2794.8	292.0	9.9964	1.6	33.6677	43.6641	225.545
2004	1370.3	4890.3	6260.6	17321.3	3920.5	228.3	3176.7	4127.3	-950.6	7.9112	1.7	28.2327	36.1439	159.689
2005	1525.9	2695.1	4221.0	22270.0	5547.5	1159.4	4255.7	5348.6	-1092	6.8519	8.8	12.1018	18.9537	76.088
2006	1753.3	451.5	2204.7	28662.5	5965.1	854.8	3548.7	6040.8	-2492	6.1169	6.7	1.57509	7.69201	36.9603
2007	2169.6	438.9	2608.5	32995.4	5727.5	125.9	5967.4	5615.2	352.2	6.5756	1.0	1.33016	7.90574	45.5439
2008	2320.3	523.3	2843.6	39157.9	7866.6	80.7	5629.0	7861.6	-2232	5.9255	0.7	1.33626	7.26178	36.1473
2009	3228.0	590.4	3818.5	44285.6	4844.6	111.6	6850.8	4475.1	2375.7	7.2891	0.8	1.33326	8.62238	78.8193
2010	4551.8	689.8	5241.7	54612.3	7303.7	187.0	9233.7	6889.7	2344.0	8.3348	1.3	1.26316	9.59795	71.7675
2011	5622.8	896.8	6519.7	62980.4	11116.8	80.0	11057.4	11296	-238.6	8.9279	0.5	1.42401	10.3519	58.6469
2012	6537.5	1026.9	7564.4	71713.9	10654.7	208.5	10207.2	10379	-172.0	9.1161	1.3	1.43194	10.5481	70.9959
2013	7119.0	1387.3	8506.3	80092.6	9759.8	77.2	10840.4	8949.4	1891.0	8.8884	0.5	1.73216	10.6206	87.1567
2014	7904.0	1631.5	9535.5	89043.6	10068.9	713.7	11403.0	9484.2	1918.8	8.8766	4.5	1.83225	10.7088	94.7033
2015	8837.0	2111.5	10948.5	94145.0	6912.5	308.1	10532.2	7231.6	3300.6	9.3866	1.6	2.24283	11.6294	158.387
2016	11058.2	3478.9	14537.1	101489.5	5616.4	631.8	9725.9	5693.8	4032.1	10.896	2.5	3.42785	14.3238	258.833
2017	12589.5	5787.5	18377.0	113711.6	7445.0	1079.1	13472.7	6863.7	6609.0	11.071	3.5	5.08964	16.1611	246.837
2018	12774.4	7759.2	20533.6	127736.8	9551.8	1641.6	16452.3	10070	6381.4	10.001	5.4	6.07436	16.0749	214.971
2019	14272.6	9022.4	23295.1	144210.5	10262.3	1573.6	18338.3	11218	7120.2	9.8971	5.1	6.25642	16.1535	226.996

Source: WEO (2021) available at https://www.imf.org/external/pubs/ft/weo/2018/01/weodata/download. aspx Note: Fisc. Def stands for fiscal deficit. Debt Serv = Debt service on external debt, total (TDS, current billions US$). Fiscal deficit is operationally defined as the general government's total expenditure less general government's total revenue. A negative deficit implies fiscal balance.



Source: WEO (2021); CBN Bulletin (2019)

Fig 1: Debt and Debt sustainability ratios in Nigeria (1981-2019)

As shown in Panel 1 of Figure 1, external debt stock (N'billion) consistently surpassed domestic debt between 1981 up to 2005. However, a reversal came with the 2005 debt forgiveness. In the aftermath of the debt cancellation in 2005, the adherence to fiscal rules in line with the medium term expenditure framework and the fiscal consolidation effect made it possible to achieve lower deficit and debt until the 2008/09 financial crisis stuck the world economies. More precisely, starting from 2006, domestic debt began to surpass external debt. Markedly, the dominance of domestic debt stock over the external debt has continued until 2019. In sympathy to the dominance of domestic debt following the 2005 debt relief, total debt stock has grown continuously from 2006 till 2019.

Considering the debt/GDP ratio, however, Panel 2 (Figure 1) shows that total external debt as a percentage of GDP grew systematically from about 1.6 percent in 1981 to 59.7 percent in 1990 before reaching its zenith of 59.8 percent in 1992. However, there was a subsequent gradual reduction in the ratio from its level of 12 percent in 2005 to about 1.5 percent in 2006, following the completion of the debt cancellation package in 2005. In 2015, the external debt/GDP ratio stood at 1.8% and the ratio has further increased in more recent times with the country posting external debt/GDP levels of 2.2 percent in 2016, 3.4 percent in 2017, 5.1 percent in 2018, and 6.1 percent in 2019.

As with external debt, domestic debt stock was bullish during the 1980s (1981-1989) while there was a steady decline during the two periods 1991-1997 and 1999-2007. Total debt as a percentage of GDP remained at a single-digit during 2006 – 2010. Recently, however, total debt has increased above 10% beginning from 2015 and has hovered between 10 % and 18% over the period 2015-2019. Between 1981 and 2019, there were about 13 episodes in which the total debt to GDP ratio exceeded the international threshold of 30 percent (Panel 2; Fig. 1).

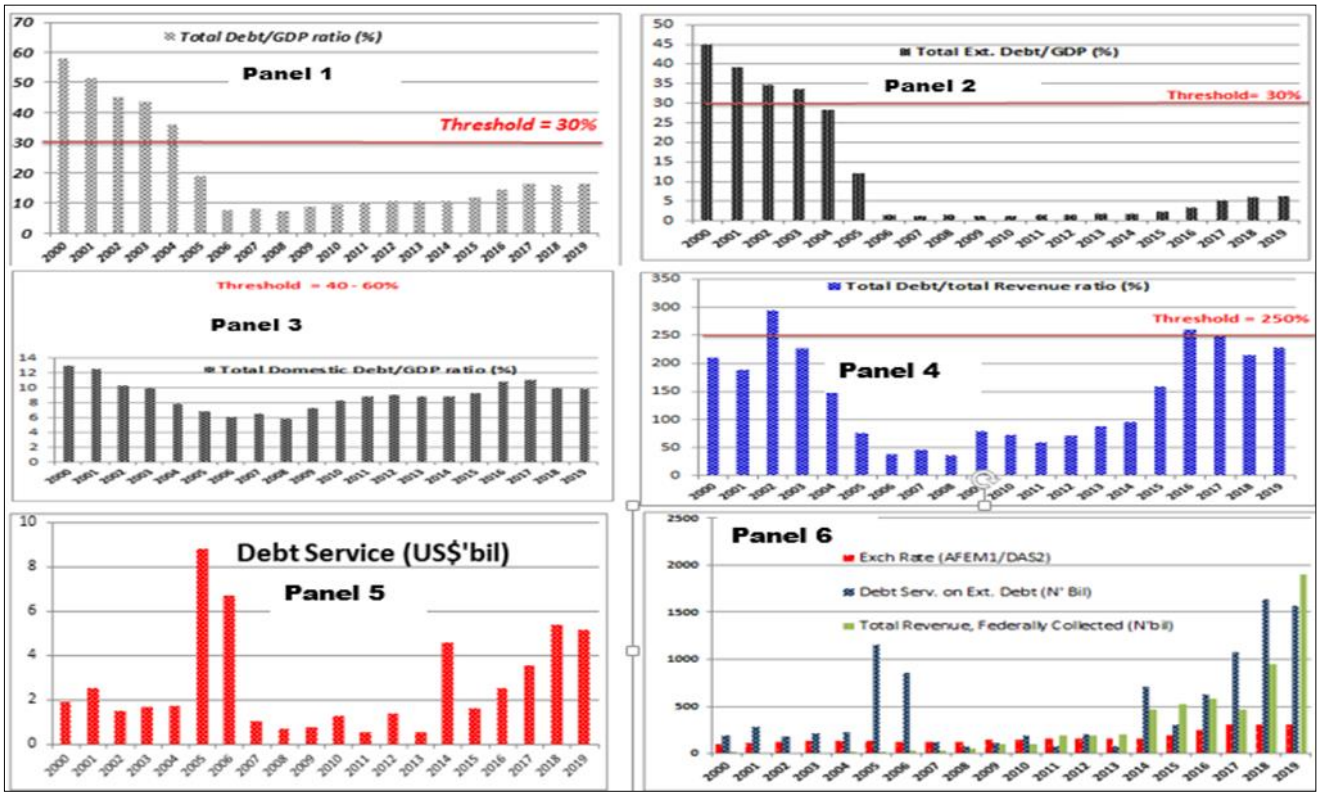
Nonetheless, notwithstanding the size of Nigeria's public debt, a critical empirical question for the country as with every other economy relates to the determination of threshold or optimal debt position (in the economic relation between output and debt) point beyond which more accumulation of debt becomes deleterious to economic growth. Such a threshold position presupposes a non-linear relationship between output growth and public debt that is determined via robust empirical investigation for Nigeria.

1.2. Nigeria's Debt Sustainability

Mostly, Nigeria's debt profile has remained within internationally acceptable limits since the external debt forgiveness of 2005 (Fig. 1). However, in 2002, the ratio of total debt to total federally collected revenue was 290.0 percent which was substantially above the international threshold of 250 percent (panel 4 of Figure 1). Total debt service has grown astronomically in the aftermath of the global economic/financial crisis from US$ 0.7 billion in 2008 to US$ 4.5 billion in 2014. More recently, Nigeria's debt service cost rose from US$ 2.5 billion in 2016 to US$ 3.5 billion in 2017 and by 2019 debt service cost stood at US$ 5.1 billion slightly lower than the 2018 debt service cost of US$ 5.4 billion.

The federal government's fiscal operations in 2002 resulted in an overall fiscal balance of N152.7 billion and total debt stock as a percentage of total federally collected revenue was 294 percent (Table 1 in Appendix). A year later (2003), Nigeria recorded a fiscal deficit of N292 billion with debt/revenue declining to 225 percent. In the aftermath of the global financial crisis, Nigeria recorded a whopping fiscal deficit of N 2.37 trillion and N 2.34 trillion in 2009 and 2010, respectively. Corresponding to the large deficit, the debt/revenue ratios declined to 78 percent and 71 percent in 2009 and 2010, respectively.

In the ensuing two consecutive years, slight relief came with the government fiscal operations resulting in fiscal balances of N235 billion and N 172 billion in 2011 and 2012, respectively. Since 2013 (to 2019) Nigeria has not only operated fiscal deficit but these deficit has shown persistently increases. Expectedly, this has also been reflected in the persistent rise in debt/revenue ratio over the same period (2013- 2019) partly because debt is the accumulation of annual fiscal/budget deficits.



Source: IMF (2021); CBN (2019); WDI (2021)

Fig 2: Nigeria's Debt Sustainability Indicators (2000-2019)

2. Brief literature

While debt accumulation could be detrimental to growth, there are reasons why countries - developed and undeveloped - have continued to accumulate large public debt. Countries borrow for, at least, two key reasons. First, one rationale for such debt accumulation is macroeconomic. Here, higher debt accumulation could be allowed to fund high levels of consumption and investment. That is, external debt are created from government's need to borrow in order to finance budget deficits (Sutherland, Hoeller, Merola, and Ziemann, 2012:9) [19]. Second, countries employ public debt as a channel to reduce their budget deficits and implement economic projects that improve the citizenry's standard of life while also fostering long-term growth and development. Public debt also boosts resource productivity by increasing output, which boosts a country's Gross Domestic Product (GDP) growth.

It is fairly well established that the effect of public debt on economic growth could have a turning point or threshold. On this, Reinhart and Rogoff (2010) [16], Reinhart and Rogoff, (2012), Wright and Grenade 2014 [22]; Siddique and Malik (2001), and Kumar and Woo (2010) [10] did studies for some advanced and emerging economies while Ekperiware and Oladeji (2012) [4], and Omotosho *et al* (2016) [11] conducted studies for Nigeria.

The study by Carmen Reinhart and Kenneth Rogoff in 2010 [16] is the pioneering and, indeed, one of the most influential paper regarding debt and growth (see Reinhart and Rogoff, 2010) [16]. Their study relied on a panel of 3700 annual observations of 44 countries spanning about two hundred years from 1790 to 2009. The same authors classified the regimes into four categories: advanced economies with low national debt to GDP ratio (below 30%), medium-low national debt to GDP ratio (between 30% and 60%), medium-high national debt to GDP ratio (between 60% and 90%), and high national debt to GDP ratio (above 90%). Their key conclusion is that countries with high public debt have significantly lower growth rate. More recent studies have used econometric and multivariate regression to control endogeneity, reverse causality, and cross-country heterogeneity that Reinhart and Rogoff, (2010) [16] appears to have sidelined. That is, notwithstanding the fact that the large span of data enables readers to have panoramic view, the inability of Reinhart and Rogoff, (2010) [16]'s model to incorporate country- specific factors such as political system and trade openness leads to the issue of heterogeneity and this has inspired sub-sequent researches

More generally, while some theoretical literature concludes a negative relationship between public debt and economic growth; others conclude a positive relationship. Yet, other literature argues that the relationship is uncertain and remains an empirical question and could have a turning point. And so, a logical question to ask is, what becomes the debt threshold level of debt - if any - above which debt becomes a burden on growth.

Favour, Ideniyi, Oge, and Charity (2017) investigated the extent to which external debt has an impact on Nigeria's economic output, as well as whether internal debt has a

substantial impact on national output in Nigeria. The study's findings revealed that external debt has a significant negative influence on economic growth, and that domestic debt (DMD) has a strong negative link with economic growth throughout the period under discussion. In light of the findings, the government should minimize external debt and ensure that the funds received are used conscientiously.

The nexus between government debt and economic growth in Nigeria was also studied by Oyedele, David, and Omojola (2016) [13]. The study's findings revealed that there is no long-term association between Nigeria's governmental debt and economic growth. Furthermore, the study indicated that per capita internal public debt has a positive but non-significant link with economic growth, whereas per capita external public debt has a negative but non-significant relationship with economic growth. Following these findings, the study concludes that the government should be more committed to the people's well-being by distributing borrowed monies to high-yielding sectors.

Idris and Ahmad (2017) [9] examined the productivity of governmental debt borrowing and economic growth in the Sub-Saharan African region. The findings revealed that domestic debt has a detrimental impact on economic growth. The study recommended that fiscal policy practitioners and other related policymakers pay undivided attention to the productive utilization of any domestic borrowed funds and ensure that resources are assigned to specific prospective growth programs and that adequate loan-repayment capacities are well-established as a matter of urgency.

An evaluation of the literature reviewed indicates that the question of what constitutes the optimal or threshold debt remains an empirical question and can, in fact, in the country-specific context. There is also scanty literature that has attempted to examine possible threshold effects in the economic relationship between Nigerian public debt provide and economic growth. Thus, from the review of theoretic literature, a logical empirical question to ask is what constitutes the debt threshold, if any, above which debt becomes a burden on growth. This paper makes a unique contribution to the literature by providing new empirical evidence on public debt and economic growth nexus, and by examining the possible threshold effect of public debt types on economic growth in Nigeria. The result is robust as it controls for omitted variable bias and reverses causality.

3. Methodology and Data

This section first lays out the data sources and variable definitions (Table 3) before outlining the empirical isometric deployed for the estimation of the dynamic threshold regression analysis.

3.1. Model Specification

The empirical threshold regression model that is specified follows closely those of Doguwa (2012) [3], Proano, Schoder and Semmler (2014), and Omotosho *et al* (2016) [11] which has been elaborated in the ensuing literature by Zaghdoudi T (2019) [23] and Hu (2019) [8], among others. As in Hu (2019) [8], GDP growth rate (GDPG) is the key dependent variable in this paper. The two core explanatory variables are the debt/GDP ratio (d_t), and the indicator (dummy) variable regarding whether or not one lag of the realized Nigeria's debt/GDP ratio (d_{t-1}) exceeds some threshold value (d*) i.e., $I(d_{t-1} > d^*)$. Government size and population growth rate are included as control variables (see Table 3). The estimable regression model from which to calculate the threshold level of debt is obtainable via the steps that follow in the sequel, thus:

Let the GDP growth rate be expressed as a function of the dummy variable for excess debt, $I(d > d^*)$, as follows:

$$GDPG_t = \lambda_0 + \lambda_1 I(d_t > d^*) + \varepsilon_t \qquad 1$$

Where $GDPG_t$ is the annual growth rate of GDP; λ_0 is a constant; $I(d_t > d^*)$ is an indicator (dummy) variable which is equals 1 if the debt/GDP ratio is above the debt threshold and 0 otherwise. d* is the threshold debt/GDP ratio to be estimated (d* refers to different debt.GDP values used for the iteration process in the search for optimal debt threshold). ε_t is the residual. Following the standard approach as in Hu (2019) [8], equation 1 is augmented to incorporate a second order polynomial term (d_t^2) as follows:

$$GDPG_t = \eta_0 + \eta_1 d_t + \eta_2 d_t - d_{t-1} + \eta_3 d_t^2 + \varepsilon_t \qquad 2$$

Where d_t is the current debt-to-GDP ratio; $d_t - d_{t-1}$ is the Differenced d_t; d_t^2 is the Squared d_t; and η_0 is the constant. d_t^2 is included to capture possible non-linear effect of debt on growth and it is subsequently employed here in the calculation of the debt threshold using the alternative procedure as specified in equation 5; all other variables are as earlier defined.

As can be easily seen, equation 1 and 2 could suffer two key estimation problems: (i) reverse causality: This might occur when the dependent variable (GDPG) also impact the explanatory variable (debt). To deal with such bi-causality, instrumentation using one year lagged per capita GDP ($GDPC_{t-1}$) is employed. It is notable that while the use of one lag of GDPC as an instrument has the advantage of providing easily interpretable result, it does not completely eliminate reverse causality. To account for possible non-linear effect of debt, the squared one year lag of debt/GDP ratio (d_{t-1}^2) is also included in the final estimable equation; (ii) omitted variable bias: This can occur when the regression equation excludes a variable that is correlated with other covariates and the dependent variable. To deal with omitted variable bias, our final estimable equations (eq. 3 and 4) incorporate population growth rate, lagged GDP growth rate, and government size (measured as total government expenditure-to-GDP ratio) as control variables.

A rationalization of the choice of the variables is in order. Including lagged GDP growth rate is important to account for possible multi-collinearity between the dependent variable (GDP growth rate) and one of the explanatory variable (lagged debt/GDP). In particular, if current GDP growth rate and current debt/GDP ratio are negatively correlated, then excluding lagged debt/GDP ratio will lead to downward bias on the coefficient for debt/GDP ratio. As with standard practice, it is notable that the selection of optimal lag in the model is based on minimum Aikike Information Criterion (AIC).

Given the above formalization, this study relies on two estimation approach – namely; the OLS estimation (of

equation 3) to retrieve the threshold debt/GDP level through an interative process for 73 different values of d*, and the OLS estimation of a polynomial function (eq4). The first estimation equation (eq 3) for the threshold value of debt relies on two key procedures for the choice of the threshold debt, namely. (i) coefficient method – here one simply choose a debt level (d*) that produces a 'coefficient' such that any other debt level greater than such debt (d*) will produce a negative coefficient and any other debt level lower than such debt (d*) will produce positive coefficient). In more specific terms, we estimate the coefficients for indicator variable ($I(d_{t-1} > d^*)$) obtained for all possible integer values of total debt/GDP ratios from 7 (minimum debt/GDP ratio) to 80 (maximum debt/GDP ratio) and pick the debt/GDP ratio corresponding to the point at which continues decline sets in (or a sign swith from positive to negative sets in) for the indicator variable. This becomes the threshold debt value for the Nigerian economy. When combined with the coefficient that produces minimum residual sum of square (RSS), this gives the *optimal d** for the Nigerian economy from all possible d* that are considered in the iteration process.

The variable d*, and by extension the indicator variable $I(d_{t-1} > d^*)$, is the variable used for the iteration process in the search for optimal debt threshold (optimal d*). Excess debt, $(d_{t-1} > d^*)$, is operationally defined here as the debt level over and above the threshold level of debt. As noted, the threshold debt/GDP ratio so obtained represent the debt level above which a continual decline (and/or sign reversal) of the coefficient of the inicator variable (calculated from debt/GDP) sets in. In this regard, our method entails two estimation approaches, namely: i) re-estimating equation 3 for 73 sets of d*, i.e., implying the various values of the indicator variable ($I(d_{t-1} > d^*)$ obtained for every integer value of debt/GDP ratio between 7 and 80 and picking the d^* that produces a declining (or reversal of sign of) coefficient of the indicator variable (see Hu, 2019:10) [8].; and (ii) minimum RSS method. Here, we simply find the threshold value of debt that minimizes the residual sum squared (RSS).

Conversely, the second estimation equation (eq.4) entails the implementation of a growth model that is polynomial in lagged debt-to-GDP growth rate (see Hu, 2019:9; Omotosho, Bawa, and Doguwa, (2016:15) [8, 11].

The first estimation equation is specified thus,

$$GDPG_t = \alpha_0 + \alpha_1 d_{t-1} + \alpha_2 GDPCG_{t-1} + \alpha_3 POPG_t + \alpha_4 Gsize_t + \alpha_5 I(d_{t-1} > d^*) + U_t \quad 3$$

where $GDPG_t$ is the annual growth rate of GDP; α is a constant; d_{t-1} is a vector of one year lag of the debt-to-GDP ratio; $I(d_{t-1} > d^*)$ is a vector of an indicator (dummy) variable which is equals 1 if the one year lagged debt/GDP ratio is above the debt threshold and 0 otherwise and it is included to capture the threshold growth effect of public debt/GDP ratio; $POPG_t$ is the population growth rate at time t; and $Gsize_t$ is the size of government at time t (approximated by total government expenditure to GDP ratio; and *Ut* is a vector of residuals assumed to have zero mean and distributed as $N(0, \Sigma)$.

The second estimation equation is specified as follows:

$$GDPG_t = \beta_0 + \beta_1 d_{t-1} + \beta_2 d_{t-1}^2 + \beta_3 GDPCG_{t-1} + \beta_4 POPG_t + \beta_5 Gsize_t + \beta_6 I(d_{t-1} > d^*) + U_t \quad 4$$

where: d_{t-1}^2 is the square of the vector of one year lag of the debt-to-GDP ratio; and all other variables are as earlier defined in equation 3. It is notable that for equation 3, the threshold debt is obtained by using a combination of minimum RSS and picking the d^* at which 'continual decreasing' coefficient sets in or reversal of the sign of the indicator variable from positive to negative sets in)

For equation 4, the threshold debt is obtained by calculating the turning point from the coefficients of the first lag of debt/GDP ratio (d_{t-1}) and its square (d_{t-1}^2). In particular, the turning point is obtained as follows:

Solution to a polynomial of degree two; turning point=

$$-\frac{\beta_1}{2 \times \beta_2} \quad 5$$

Where β_1 is the coefficient of d_{t-1} and β_2 is the coefficient of d_{t-1}^2.

Table 2: Data Sources and Variables Descriptions

Variable	Code	Description	Source
Dom	Dom	Total Domestic Debt (N'bil)	CBN 2019
Ext	Ext	Total External Debt (N'bil)	CBN 2019
TD	TD	Total Debt (N'bil)	CBN 2019
GDP	GDP	Nominal GDP (N'bil)	CBN 2019
TFCR	TFCR	Total Federally Collected Revenue (N'bil)	CBN 2019
DS	DS	Total Debt Service cost	WDI (2021)
EXP	EXP	General Government total exp ((N'bil))	CBN 2019
DEF	Def	Fiscal Deficit (N'bil)	WEO (2021)
REV	REV	General Govt. Tot Rev	WEO (2021)
DY	DY	Domestic Debt to Nominal GDP ratio (%)	CBN 2019
EY	EY	External Debt to Nominal GDP ratio (%)	CBN 2019
TY	TY	Total Debt to Nominal GDP ratio (%)	CBN 2019
GDPG	GDPG	GDP Growth rate (%)	WDI (2021)
POGG	POGG	Population Growth rate (%)	WDI (2021)
GSIZE	GSIZE	Government Size (Govt. Expen. to GDP ratio) (%)	CBN 2019
Debt_Rev	Debt_Rev	Total debt to total revenue ratio (%)	CVBN 2019

4. Analysis and Discussion of Results

For the first estimable equation (eq.3), we run regression for every integer value between 7 (minimum total debt/GDP ratio) and 80 (maximum debt/GDP ratio) as the threshold value (d^*) and obtained coefficient corresponding to each $I(d_{t-1} > d^*)$ (Table 3).

Table 3: Regression Result of Equation 3

Expl.Variable	d*=36	d*=37	d*=38	d*=39	d*=40	d*=41	d*=42	d8=43	d*=44	d*=45
dt-1	3.2	2.5	2.4	2.1	1.9	1.53	1.77	1.82	2	2.1
	(0.11)	(0.23)	(0.47)	(0.66)	(0.14)	(0.07)	(0.87)	(0.86)	(0.22)	(0.31)
GDPCG $_{t-1}$	2.8	2.5	2.3	2.3	1.5	0.87	0.69	0.74	0.6	0.71
	(0.14)	(0.23)	(0.65)	(0.001)	(0.03)	(0.04)	(0.42)	(0.87)	(0.13)	(0.32)
$POPG_t$	1.9	1.7	1.1	1.2	0.12	0.62	0.71	0.69	0.34	0.61
	(0.18)	(0.02)	(0.55)	(0.21)	(0.03)	(0.41)	(0.9)	(0.37)	(0.6)	(0.2)
$gsize_t$	4.2	3.8	13	3.9	2.7	1.98	2.6	1.67	2.8	3.0
	(0.61)	(0.54)	(0.51)	(0.71)	(0.21)	(0.003)	(0.54)	(0.67)	(0.33)	(0.22)
$I\left(d_{t-1} > d^*\right)$	3.9	3.7	1.4	3.4	1.1	-0.34	-1.45	-2.4	-2.5	-2.8
	(0.89)	(0.88)	(0.02)	(0.22)	(0.34)	(0.04)	(0.45)	(-0.98)	(0.4)	(-0.7)
Residual Sum of Square (RSS)	466	458	454.2	454.13	453	452	460	480	484	490

Note: Decision Rule 1: pick the d^* at which continual decreasing coefficient sets in (or reversal of the sign of the indicator variable from positive to negative sets in) - corresponding to a minimum RSS (see Hu, 2019 [8]:9; Omotosho, Bawa, and Doguwa, (2016:15) [11]. The figures in parenthesis are the probability values. The dependent variable is GDP growth rate. The main explanatory variables of interest here are the one year lagged debt-to-GDP ratio and squared one year lagged debt –to-GDP ratio. While we estimated all possible integers between the minimum (7) and maximum value of debt (80) (see deceptive statistics of data), only the results for d* that lies few integers to the left and right of the turning point (obtained as 41 %) are reported here, i.e., only estimated results for d* at 36, 37, 38, 39, 40, 41, 42, 43, 44, and 45 are reported due to space factor.

From the full set of result, it is easily seen that the coefficients have a decreasing trend (as threshold values d^* increase). From the full set of results (for d^*=7 through d^* =80), the coefficient of the indicator variable, i.e., $I\left(d_{t-1} > d^*\right)$, increases initially before a decline sets in and it eventually become negative (see Table 5 for a summary).

The coefficient decreased from debt/GDP of 36 % from positive to negative at debt/GDP ratio of 41% and further decrease as threshold value increase. Nonetheless, the coefficient become less negative when threshold value increase to 60%. The coefficient of $I(d_{t-1} > d^*)$ that corresponds to (d^*) = 41% is -0.34. And the coefficient is statistically significant at 5% level. This seems to suggest a robust result that all things being equal, Nigeria with a debt threshold value of 41% would experience a 0.34% decline in growth than with debt-to-GDP ratio below the threshold. This is study relies on "the coefficient method" because the "minimizing RSS" method has been shown to imply an optimization method with less economic interpretation (Hu, 2019) [8].

Zooming in on the second estimation equation (eq.4), one year lagged debt to GDP ratio and squared one year lagged debt to GDP ratio are the key variables of interest (Table 4). And the decision rule is to use equation 5 to calculate the turning point.

Table 4: Regression Result of Equation 4- Polynomial Function

Regressors	d*=36	d*=37	d*=38	d*=39	d*=40	d*=41	d*=42	d*=43	d*=44	d*=45
d_{t-1}	1.1	0.98	0.77	0.71	0.51	0.23	0.45	0.66	0.64	0.69
	(0.87)	(0.45)	(0.02)	(0.86)	(0.51)	(0.03)	(0.18)	(0.15)	(0.23)	(0.13)
$d_{t-1}^{\ 2}$	-0.99	-0.79	-0.728	-0.98	-0.72	-0.0029	-035	-7.6	-7.9	-8
	(0.63)	(0.22)	(0.88)	(0.43)	(0.93)	(0.02)	(1.2)	(0.08)	(0,9)	(0.45)
GDPCG $_{t-1}$	3	3.1	3.5	1.9	2.31	0.39	0.47	0.82	1	1.82
	(0.98)	(0.5)	(0.63)	(1.0)	(0.98)	(0.01)	(0.55)	(0.75)	(0.33)	(0.21)
$POPG_t$	3.5	3	2.8	1.8	2	3	7	4.1	9.3	10.5
	(0.4)	(0.3)	(0.32)	(0.3)	(0.32)	(0.19)	(0.56)	(0.99)	(0.34)	(0.11)
$gsize_t$	2.5	2.1	2.1	2.4	2.1	1.6	3.4	2.7	3.4	5.7
	(0.32)	(0.11)	(0.22)	(0.71)	(0.02)	(0.67)	(0.39)	(0.71)	(0.3)	(0.31)
$I(d_{t-1} > d^*)$	1.9	2.2	2.8	2.4	1.8	3.1	3.3	4.7	3.9	5.3
	(0.21)	(1.3)	(0.65)	(1.1)	(0.34)	(0.79)	(0.09)	(0.28)	(0.2)	(0.19)

Note: Decision Rule 2: Use equation 5 to calculate the turning point, ie., use $-\frac{\beta_1}{2\times\beta_2}$. The figures in parenthesis are the probability values. The dependent variable is GDP growth rate. The main explanatory variable of interest is the indicator variable regarding whether or not the debt/GDP ratio lies above the threshold, $I(d_{t-1} > d^*)$. As with equation 3, while all possible integers integers between the minimum (7) and maximum value of debt (80) are estimated (see decriptive statistics of data), only the results for d* that lies few integers to the left and right of the turning point of 41 are reported here, i.e., only estimated results for d* at 36, 37, 38, 39, 40, 41, 42, 43, 44, and 45 are reported due to space factor. The coefficients of these variables are inputted into equation 5 to Obtained the threshold debt-to-GDP ratio of 41% using equation 5.

The estimated coefficient for the two variables, d_{t-1} and $d_{t-1}^{\ 2}$, are 0.23 and -0.0029 respectively. Both coefficients are significant at 5% level. This suggest that a 1% increase

in debt to GDP ratio will result in a 0.126% increase in economic growth rate in the following year. However, because of the coefficient for squared one year lagged debt to GDP ratio is negative, the positive effect of debt to GDP ratio on economic growth rate decreases as debt increase. The turning point – earlier obtained via decision rule one - can be calculated using equation 5 which gives an estimate for the threshold value of 41%, thus.

Solution to a polynomial of degree two; turning point =

$$-\frac{\beta_1}{2\times\beta_2}$$

Where β_1 is the coefficient of d_{t-1} and β_2 is the coefficient of d_{t-1}^{2}.

$-\frac{\beta_1}{2\times\beta_2}$ = -(0.237628/(2(-0.002869)= -(0.237628/-0.005738) = 41.4%.

As can be easily seen, both regressions give similar estimations for the threshold value which is lower than 90% found by Reinhart and Rogoff (2010) [16] for advanced economies. This threshold is also lower than the total public debt (percentage of GDP) threshold of 73.7 percent found by Omotosho *et al* (2016) [11] for the Nigerian economy.

6. Conclusion and Lessons for policy

We found clear evidence that the Nigerian optimal debt threshold might have hitherto been largely and generally mis-reported. Using a combination of two robust approaches, namely; the estimation of a polynomial function of debt/GDP ratio and an iteration process in which over 80 OLS equations were estimated to arrive at an optimal debt threshold , it is found that the turning point for the Nigeria's total public debt ratio is about 41 percent of GDP. In particular, the study relied on two decision rules – each for the two estimable regression equations (eq. 3 and eq. 4). The decision rule for equation 3 utilizes combination of the debt/GDP (d^*) at which continual decreasing coefficient sets in (or reversal of the sign of the indicator variable from positive to negative sets in) and a minimum residual sum of square (RSS) as in Hu 2019: 9) [8]. Conversely, the decision rule of equation 4 relies on the estimates of the beta coefficient from whxih the turning point is calculated. Both approaches indiacted a debt/GDP threshold of about 41 percent.

Comparatively, higher debt threshold for advanced economies like the United States and Japan (at about 90percent)is not too surprising given that such economies are not only more diversified and technologically advanced , but also their debt are asset-based and asset-backed. This is why a country like Japan – although being among the most indebted countries in the world - remains credit worthy in the international community.

In particular, the finding of a debt-to-nominal GDP threshold of about 41% indicates that debt level above this value could have deleterious effect on economic growth in Nigeria. The policy import of this finding is the need for government to exercise caution in further debt accumulation. The absence of such clear and conscientious effort to cut public debt would not policy misalign any fiscal consolidation efforts but would clearly be inconsistent with the country's long run growth objective.

Due to the dearth of consistent budget data (over the entire sample period), we use the total fiscal deficit (rather than budget deficit). The fiscal deficit (general government) is defined as the fiscal position of government after accounting for capital expenditure. Operationally, it is obtained as general government's total expenditure less general government's total revenue. While total fiscal balance, is statistically defined as the net lending of the general government (revenue less expenditure), the total fiscal deficit is defined as the excess of expenditure over revenue (expenditure less revenue). In this sense, if the sign of the total fiscal deficit is negative, it is called total fiscal balance which implies net lending of the general government. As Mourre *et al*. (2013) note, government balance according to the EDP (excessive deficit procedure) definition is the net lending (surplus) / net borrowing (deficit) of the general government. The concept of total fiscal deficit as clearly defined here differs from the concept of 'primary deficit' which is simply total fiscal deficit less interest payment.

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